Production function and perfect competition

$$Q_x(K;L) = K^2 L^{0.5}, \qquad w = 8; r = 5$$

1. Consider $K = 2$. Obtain the cost function $C(Q)$ and calculate the marginal cost and the average cost.
2. Obtain the value of Q and P that would occur in perfect competition with the condition that profits are zero.

Solution

1. Considering that capital is fixed, the production function is:

$$Q = 4L^{0.5}$$

The firm's costs are:

$$C = 8L + 5 * 2 = 10 + 8L$$

Clearing L from the production function:

$$\frac{Q^2}{4^2} = \frac{Q^2}{16} = L$$

Inserting the value of L to obtain the cost function:

$$C(Q) = 10 + 8\frac{Q^2}{16} = 10 + \frac{Q^2}{2}$$

Therefore the marginal cost function:

$$\frac{dC}{dQ} = CMC = Q$$

The average cost function:

$$AC = C/Q = 10/Q + Q/2$$

2. In perfect competition, the price must be such that the marginal cost intersects the average cost:

$$10/Q + Q/2 = Q$$

$$10/Q = Q/2$$

$$20 = Q^2$$

$$\sqrt{20} = Q$$

So the price is $P = \sqrt{20}$, this makes the profits zero:

$$\pi = R - C = P \cdot Q - (10 + Q^2/2) = \sqrt{20} \cdot \sqrt{20} - (10 + \sqrt{20}^2/2) = 0$$